Universal Space of Multidimensional Infinitesimal Points: Measure, Integration

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Classical science [1] based on the real numbers without namely actual infinities and infinitesimals and on at most countable number operations cannot resolve Zeno's paradox (5th century BC) on dividing an object of finite measure $M > 0$ into an infinite set of equal parts of measure m: if $m = 0$, then $M = 0$; if $m > 0$, then $M = +\infty$ (heap of infinities without their differentiation). Zero-measure (0M) and zero-dimensional (0D) points cannot compose 1D lines, 2D surfaces, and 3D spatial bodies. Kepler's and Cavalieri's composing an area of intervals, a volume of areas, and especially a circle of central triangles whose limits are radii have no justification because of 0M and 0D points. Integration has no point-wise summation nature, is artificial, only potentially infinitesimal, and hence zero-measure fully nonsensitive without conservation law universality. For example, $\int_a^b f(x)dx$ does not depend on including or excluding zero-measure endpoints a and/oder b. No common measure M_n is universal. The linear M_1, area M_2, and volume M_3 measures are only finitely sensitive to bounded parts of lines, surfaces, and the space. No common measure holds for mixed dimensions.

Unimathematics [2-5] based on uniphilosophy [2, 6] and metauniphilosophy [2, 7] is perfectly sensitive and exactly measures and integrates namely actual infinities with conservation law universality by actually infinitesimal differences. Quantisets with element quantities q, uninumbers, also uncountable operations, and uniquantities Q as counting point unimeasures discover actually infinitesimal point measure $Q_n = Q/\Omega^n$ and point-wise space nature $\Pi_{j=1}^n|x_j-0.5/\Omega,\ x_j+0.5/\Omega|$ of half-open/closed point $({}_{j=1}{}^n x_j)$ (in n-dimensional Euclidean space R^n) for which $Q({}_{j=1}{}^n x_j) = 1$, $Q_n({}_{j=1}{}^n x_j) = 1/\Omega^n$ using countable cardinality $\omega = Q\{1, 2, ...\}$ and continuum cardinality $\Omega = Q(0, 1] = Q|0, 1| = Q({}_{1/2}0 + (0, 1) + {}_{1/2}1)$. At least continually adding points or point functions gives lines, surfaces, and spaces (possibly their parts, namely open, half-open/closed, and closed intervals (segments) of length $M_1 = L$ (with $Q = L\Omega -1$, $Q_1 = L - 1/\Omega$; $Q = L\Omega$, $Q_1 = L$; $Q = L\Omega +1$, $Q_1 = L+1/\Omega$, respectively), areas, and volumes) or their functions with inventing actually infinitesimal point-wise summation integration G, see Figures 1, 2:

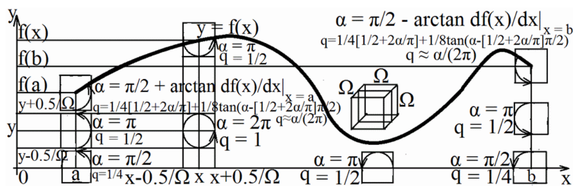

Fig.1. Point-wise space layer integration

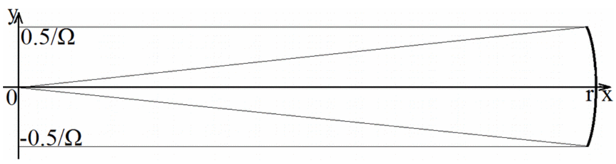

Figure 2. Actually infinitesimal point-wise circle sector-triangular integration

$$Q|a, b| = Q\Sigma_{|a,b|}|x-0.5/\Omega,\ x+0.5/\Omega| = \Sigma_{|a,b|}Q|x-0.5/\Omega,\ x+0.5/\Omega| = (b-a)\Omega\ 1/\Omega\ \Omega = (b-a)\Omega;$$
$$Q_1|a, b| = Q_1\Sigma_{|a,b|}|x - 0.5/\Omega,\ x + 0.5/\Omega| = \Sigma_{|a,b|}Q_1|x-0.5/\Omega,\ x+0.5/\Omega| = (b - a)\Omega\ 1/\Omega = b - a.$$
$$Q\Pi_{j=1}^n|a_j,\ b_j| = \Pi_{j=1}^n((b_j-a_j)\Omega) = \Omega^n\Pi_{j=1}^n(b_j-a_j);\ Q_n\Pi_{j=1}^n[a_j,b_j] = \Pi_{j=1}^n g_n[a_j,b_j] = \Pi_{j=1}^n(b_j-a_j+1/\Omega).$$

To provide complete (also uncountable) both analytic and geometric additivity without intersections and absorption, for any (also corner) point (x, y), regard its angle α (Fig. 1) namely internal for an area, use floor function $[z]$, and take $q = 1/4[1/2 + 2\alpha/\pi] +1/8 \tan(\alpha - [1/2 + 2\alpha/\pi]\pi/2)$ for a square or simply $q \approx \alpha/(2\pi)$ for the inscribed circle (for the 3D space, $q \approx \alpha/(4\pi)$). For internal point (x, y), $\alpha = 2\pi$, $q = 1$. For boundary differentiable point, e.g., $(x, f(x))$, $\alpha = \pi$, $q = 1/2$. Independently of above additivity,

$$G|-\omega,\ \omega|\times|0,\ f(x)| = \int_{-\infty}^{+\infty}f(x)dx;\ Q_{q(x,y)}[a,\ b]\times[0,\ f(x)] = \Sigma_{[a,b]\times[0,f(x)]}q(x,y) = \Sigma_{[a,b]}\Sigma_{[0,f(x)]}q(x,\ y);$$
$$G_{q(x,y)}[a,\ b]\times[0,\ f(x)] = Q_{q(x,y)}[a,\ b]\times[0,\ f(x)]/\Omega^2 = \Sigma_{[a,b]\times[0,f(x)]}q(x,y)/\Omega^2 = \Sigma_{[a,b]}\Sigma_{[0,f(x)]}q(x,\ y)/\Omega^2;$$
$$G[{}_q a,{}_r b]\times[{}_s 0,{}_t f(x)] = \int_a^b f(x)dx + [(q-1/2)f(a)+(r-1/2)f(b)+(s+t-1)(b-a)]/\Omega+(q+r-1)(s+t-1)/\Omega^2;$$
$$G|a,\ b|\times|0,\ f(x)| = G[{}_{1/2}a,\ {}_{1/2}b]\times[{}_{1/2}0,\ {}_{1/2}f(x)] = \int_a^b f(x)dx;$$

$G[a, b]\times[0, f(x)]=\int_a^b f(x)dx + [f(a)/2 + f(b)/2 + b - a]/\Omega + 1/\Omega^2$ (for above additivity, take $\alpha = \pi/2 + \arctan df(x)/dx$ at $(a, f(a))$, $\alpha = \pi/2 - \arctan df(x)/dx$ at $(b, f(b))$, see Fig. 1);
$G(a, b)\times(0, f(x)) = G[_0a, {}_0b]\times[_00, {}_0f(x)] = \int_a^b f(x)dx - [f(a)/2 + f(b)/2 + b - a)]/\Omega + 1/\Omega^2$.
$Q\{(x, y)|x^2+y^2\leq_{1/2}r^2\}=1/2\ r\Omega\ 1\ 2\pi r\Omega=\pi r^2\Omega^2$; $G\{(x,y)|x^2+y^2\leq_{1/2}r^2\}=\pi r^2\Omega^2/\Omega^2 = \pi r^2$ (Fig. 2);
$Q\{(x, y)|x^2+y^2\leq r^2\} = \pi(r\Omega+1/2)^2 =\pi r^2\Omega^2+\pi r\Omega+\pi/4$; $G\{(x,y)|x^2+y^2\leq r^2\}=\pi r^2+\pi r/\Omega+\pi/(4\Omega^2)$;
$Q\{(x, y)|x^2+y^2<r^2\} = \pi(r\Omega-1/2)^2 = \pi r^2\Omega^2-\pi r\Omega+\pi/4$; $G\{(x,y)|x^2+y^2<r^2\}= \pi r^2-\pi r/\Omega+\pi/(4\Omega^2)$.

Universal space discretization, measurement, and integration via multidimensional infinitesimal points provides intelligently solving urgent complicated problems, e.g., modeling real materials, cracks, and their propagation in aeronautical fatigue.

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